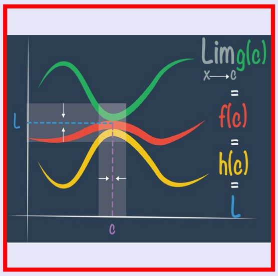
Calculus I

Lecture 10



Feb 19-8:47 AM

Class QZ 10

Given $f(x) = 3x^4 - 4x^3$ → Y-Int (0, 0)

x-Int (0,0), ($\frac{4}{3}$, 0)

1) Find $f'(x)$.

$f'(x) = \boxed{12x^3 - 12x^2}$

$= 12x^2(x-1)$

2) Solve $f'(x) = 0$.

$x^2 = 0$ $\quad$ $x - 1 = 0$

$\boxed{x=0}$ $\quad$ $\boxed{x=1}$

3) Find $f''(x)$.

$f''(x) = 36x^2 - 24x$

$= 12x(3x-2)$

4) Solve $f''(x) = 0$.

$\boxed{x=0}$ $\quad$ $3x - 2 = 0$

$\boxed{x = \frac{2}{3}}$

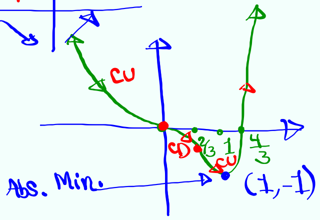
Sign chart

x	$-\infty$		0		$\frac{2}{3}$		1		∞
$f'(x)$		−	●	−		−	●	+	
$f''(x)$		+	●	−	●	+		+	
$f(x)$		↘		↘		↘		↗	

$f(1) = 3(1)^4 - 4(1)^3 = -1$

Domain $(-\infty, \infty)$

Range $[-1, \infty)$



Jan 21-7:38 AM

$f(x) = x^5 - 5x$

1) Even, $\boxed{\text{odd}}$, or neither.

$f(-x) = (-x)^5 - 5(-x) = -x^5 + 5x = -(x^5 - 5x)$
$= -f(x)$

$f(-x) = f(x) \rightarrow$ even
Sym. Y-axis

$f(-x) = -f(x) \rightarrow$ odd
$\rightarrow$ Sym origin

2) Y-Int
(0,0)

3) x-Int
$x^5 - 5x = 0$
$x(x^4 - 5) = 0$
(0,0)
$(\pm\sqrt[4]{5}, 0)$
$\rightarrow (x^2 + \sqrt{5})(x^2 - \sqrt{5}) =$
$x^2 = \sqrt{5}$
$x = \pm\sqrt[4]{5}$

4) Critical numbers

$f'(x) = 0$ or undefined
$f'(x) = 5x^4 - 5$
$= 5(x^4 - 1)$
$= 5(x^2+1)(x^2-1)$
$= 5(x^2+1)(x+1)(x-1)$

C.N. $x = \pm 1$

5) Possible inflection Points.

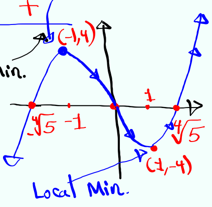
$f''(x) = 0$ or undef.
$f''(x) = 20x^3$

P.I.P. at $x = 0$

6) Sign Chart

x	$-\infty$		-1		0		1		∞
$f'(x)$		$+$	●	$-$		$-$	●	$+$	
$f''(x)$		$-$		$-$	●	$+$		$+$	
$f(x)$		↗	Max	↘		↘	Min.	↗	

Local Max



$f(1) = -4$

Domain $\rightarrow (-\infty, \infty)$

Range $\rightarrow (-\infty, \infty)$

Jan 21-8:29 AM

$f(x) = \frac{x^2}{x^2+3}$

H.A. $y = \frac{1}{1} = 1$

1) Domain $x^2 + 3 \neq 0$
$(-\infty, \infty)$

2) Show $f(x)$ is an even function

$f(-x) = \frac{(-x)^2}{(-x)^2+3} = \frac{x^2}{x^2+3} = f(x)$

Sym $\rightarrow$ Y-axis

3) Y-Int (0,0)

4) x-Int. (0,0) Twice

5) Find $f'(x)$, Find x where $f'(x)=0$ or Undef.

$f'(x) = \frac{2x(x^2+3) - x^2 \cdot 2x}{(x^2+3)^2}$

$= \frac{2x^3 + 6x - 2x^3}{(x^2+3)^2}$

$f'(x) = \frac{6x}{(x^2+3)^2}$

$\frac{6x}{(x^2+3)^2} = 0$ $x^2+3 \neq 0$
$6x = 0$
$\boxed{x = 0}$

6) Find $f''(x)$, Find x where $f''(x)=0$ or Undef.

$f''(x) = 6 \cdot \frac{1(x^2+3)^2 - x(2(x^2+3)^1 \cdot 2x)}{[(x^2+3)^2]^2} = \frac{6[(x^2+3)^2 - 4x^2(x^2+3)]}{(x^2+3)^4}$

$= \frac{6\cancel{(x^2+3)}[x^2+3-4x^2]}{(x^2+3)^{\cancel{4}3}}$

$= \frac{6(3-3x^2)}{(x^2+3)^3}$

$= \frac{-3 \cdot 6(x^2-1)}{(x^2+3)^2}$

$f''(x) = \frac{-18(x^2-1)}{(x^2+3)^3}$

$f''(x) = 0$
$x^2 - 1 = 0$
$x = \pm 1$

undefined
$x^2 + 3 \neq 0$

Jan 21-8:47 AM

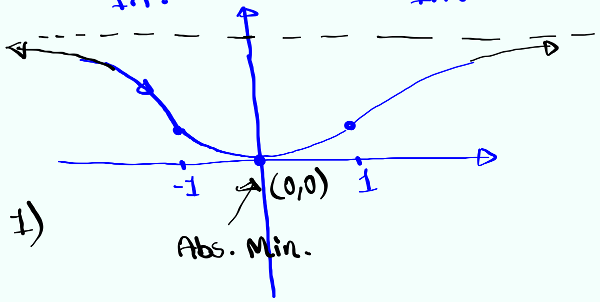
$f'(x) = \frac{6x}{(x^2+3)^2}$ $\qquad$ $f''(x) = \frac{-18(x^2-1)}{(x^2+3)^3}$

$x = 0$ $\qquad$ $x = \pm 1$

x	-∞		-1		0		1		∞
f'(x)		−		−	●	+		+	
f''(x)		−	●	+		+	●	−	
f(x)		↘	I.P.	↘		↗	I.P.	↗	

$f(-1) = \frac{1}{4}$

$f(1) = \frac{1}{4}$



Range $[0, 1)$

Jan 21-9:10 AM

$f(x) = \frac{x}{x^3-1}$ $\qquad$ $\boxed{\text{Wolframalpha.com}}$

1) Domain $x^3-1 \neq 0$
$x^3 \neq 1$
$\boxed{x \neq 1}$

2) V.A. & H.A.
$x=1$ $\quad$ $y=0$

3) Y-Int $(0,0)$

4) X-Int $(0,0)$

5) $f'(x) = \frac{-(2x^3+1)}{(x^3-1)^2}$

6) $f''(x) = \frac{6x^2(x^3+2)}{(x^3-1)^3}$

$f'(x)=0 \rightarrow 2x^3+1=0$
$x = \sqrt[3]{\frac{-1}{2}}$
$= -\sqrt[3]{\frac{1}{2}}$

$f''(x)=0 \rightarrow 6x^2(x^3+2)=0$
$x=0$ $\quad$ $x = \sqrt[3]{-2}$
$= -\sqrt[3]{2}$

$f'(x)$ is und. at
$x^3-1=0$
$x^3=1$
$x = \sqrt[3]{1} = 1$

$f''(x)$ und. at $x^3-1=0$
$x=1$

Do the Sign chart & Draw your graph

Jan 21-9:20 AM

$f(x)=\frac{x}{x^2-1}$

1) Domain
$x\neq\pm1$

2) Y-Int (0,0)

3) x-Int (0,0)

4) Continuity
everywhere except ±1

5) $f(-x)=\frac{-x}{(-x)^2-1}=\frac{-x}{x^2-1}=-\frac{x}{x^2-1}=-f(x)$

$f(-x)=-f(x) \rightarrow f(x)$ is an odd function
symmetric w/t origin.

6) $f'(x)=\frac{-(x^2+1)}{(x^2-1)^2}$

$f'(x)=0$ $x^2+1=0$
No soln.

$f'(x)$ is und. $x^2-1=0$
$x^2=1$ $x=\pm\sqrt{1}$
$x=\pm1$

7) $f''(x)=\frac{2x(x^2+3)}{(x^2-1)^3}$

$f''(x)=0$ $2x(x^2+3)=0$
↳ $x=0$

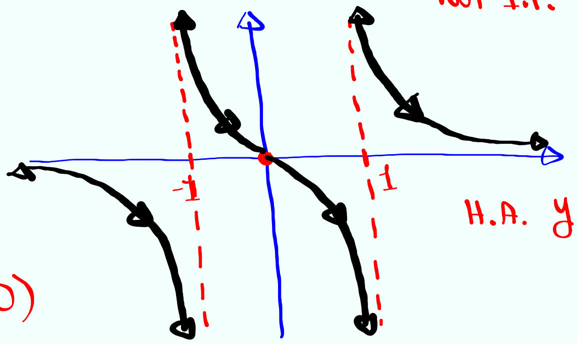
$f''(x)$ is und. at
$x^2-1=0$
⋮
$x=\pm1$

Jan 20-9:40 AM

$f(x)=\frac{x}{x^2-1}$, $f'(x)=\frac{-(x^2+1)}{(x^2-1)^2}$ $f''(x)=\frac{2x(x^2+3)}{(x^2-1)^3}$

x	-∞		-1		0		1		∞
f'(x)		−	○	−		−	○	−	
f''(x)		−	○	+	●	−	○	+	
f(x)		↘		↘		↘		↘	

Not I.P.



H.A. $y=0$

Range
$(-\infty,\infty)$

Jan 21-9:39 AM

Use chain rule to find the first derivative

1) $f(x) = \tan\sqrt{1-x}$

$$f'(x) = \sec^2\sqrt{1-x} \cdot \frac{d}{dx}\left[\sqrt{1-x}\right]$$

$$= \sec^2\sqrt{1-x} \cdot \frac{d}{dx}\left[(1-x)^{1/2}\right]$$

$$= \sec^2\sqrt{1-x} \cdot \frac{1}{2}(1-x)^{\frac{1}{2}-1} \cdot (-1)$$

$$= \sec^2\sqrt{1-x} \cdot \frac{-1}{2(1-x)^{1/2}} = \boxed{\frac{-\sec^2\sqrt{1-x}}{2\sqrt{1-x}}}$$

Jan 21-10:08 AM

2) $f(x) = \dfrac{1}{\sin(x - \sin x)}$

$$f(x) = \csc(x - \sin x)$$

$$f'(x) = -\csc(x - \sin x)\cot(x - \sin x) \cdot \frac{d}{dx}\left[x - \sin x\right]$$

$$= -\csc(x - \sin x)\cot(x - \sin x) \cdot (1 - \cos x)$$

$$= \boxed{\csc(x - \sin x)\cot(x - \sin x)(\cos x - 1)}$$

Jan 21-10:13 AM

Use implicit diff. to find $\frac{dy}{dx}$.

1) $\sin(xy) = x^2 - y$

$$\frac{d}{dx}\left[\sin(xy)\right] = \frac{d}{dx}\left[x^2 - y\right]$$

$$\cos(xy) \cdot \frac{d}{dx}\left[xy\right] = 2x - \frac{dy}{dx}$$

$$\cos(xy) \cdot \left[1 \cdot y + x \cdot \frac{dy}{dx}\right] = 2x - \frac{dy}{dx}$$

$$y \cos xy + x \cos xy \frac{dy}{dx} = 2x - \frac{dy}{dx}$$

$$x \cos xy \frac{dy}{dx} + \frac{dy}{dx} = 2x - y \cos xy$$

$$(x \cos xy + 1) \frac{dy}{dx} = 2x - y \cos xy$$

$$\boxed{\frac{dy}{dx} = \frac{2x - y \cos xy}{x \cos xy + 1}}$$

Jan 21-10:18 AM

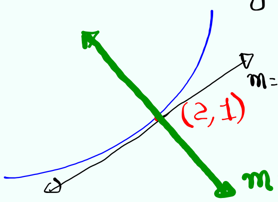
Find eqn of the normal line to the graph of $x^2 + 4xy + y^2 = \overset{\checkmark}{13}$ at $(2,1)$.

verify $(2,1)$

$2^2 + 4(2)(1) + 1^2 =$

$4 + 8 + 1 =$

$13 = 13$



$m = \frac{dy}{dx}\Big|_{(2,1)}$

$$m = \frac{-1}{\frac{dy}{dx}\Big|_{(2,1)}} = \frac{-1}{\frac{-4}{5}} = \boxed{\frac{5}{4}}$$

$$x^2 + 4xy + y^2 = 13$$

$$2x + 4\left[1 \cdot y + x \cdot \frac{dy}{dx}\right] + 2y \frac{dy}{dx} = 0$$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{5}{4}(x - 2)$

$$\boxed{y = \frac{5}{4}x - \frac{3}{2}}$$

at $(2,1)$

$$2(2) + 4\left[1 \cdot 1 + 2 \frac{dy}{dx}\Big|_{(2,1)}\right] + 2 \cdot 1 \frac{dy}{dx}\Big|_{(2,1)} = 0$$

$$4 + 4 + 8 \frac{dy}{dx}\Big|_{(2,1)} + 2 \frac{dy}{dx}\Big|_{(2,1)} = 0$$

$$10 \frac{dy}{dx}\Big|_{(2,1)} = -8 \qquad \frac{dy}{dx}\Big|_{(2,1)} = \frac{-4}{5}$$

Jan 21-10:24 AM

Use linear Approximation to estimate $\cos 61^\circ$.

$\cos 61^\circ \approx \cos 60^\circ$

$f(x) = \cos x$

$a = 60^\circ$

$f(60^\circ) = \cos 60^\circ = \frac{1}{2}$

$f'(x) = -\sin x$

$f'(60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$

From Calc. $.485$

$\cos 61^\circ \approx \boxed{.484}809....$

$\rightarrow f(x) \approx f(a) + f'(a)(x-a)$

$\cos x \approx \frac{1}{2} + \frac{-\sqrt{3}}{2}(x - 60^\circ)$

$\cos x \approx \frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{3}\right)$

$\cos 61^\circ \approx \frac{1}{2} - \frac{\sqrt{3}}{2}(61^\circ - 60^\circ)$

$= \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot 1^\circ$

$= \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\pi}{180}$

$= \frac{1}{2} - \frac{\pi\sqrt{3}}{360}$

$\approx \boxed{.484}8850053$

$.485$

Jan 21-10:32 AM

Use linear approximation to estimate $\sqrt[4]{17}$

$\sqrt[4]{17} \approx \sqrt[4]{16} = 2$

$\sqrt[4]{17} = 2.030543...$

$f(x) = \sqrt[4]{x}$

$a = 16$

$f(16) = \sqrt[4]{16} = 2$

$f(x) = x^{1/4}$

$f'(x) = \frac{1}{4} x^{-3/4}$

$= \frac{1}{4\sqrt[4]{x^3}} = \frac{1}{4(\sqrt[4]{x})^3}$

$\rightarrow f(x) \approx f(a) + f'(a)(x-a)$

$\sqrt[4]{x} \approx f(16) + f'(16)(x-16)$

$\sqrt[4]{x} \approx 2 + \frac{1}{32}(x-16)$

$\sqrt[4]{17} \approx 2 + \frac{1}{32}(17-16) = 2 + \frac{1}{32}$

$\boxed{2.03125} = \boxed{\frac{65}{32}}$

$f'(16) = \frac{1}{4(\sqrt[4]{16})^3} = \frac{1}{32}$

Jan 21-10:40 AM

$x = x(t)$, $y = y(t)$, $y = \sqrt{2x+1}$

Find $\frac{dy}{dt}$ when $x=4$ and $\frac{dx}{dt} = 3.$

Method I

$$\frac{dy}{dt} = \frac{d}{dt}\left[\sqrt{2x+1}\right]$$

$$\frac{dy}{dt} = \frac{d}{dt}\left[(2x+1)^{1/2}\right]$$

$$\frac{dy}{dt} = \frac{1}{\cancel{2}}(2x+1)^{-\frac{1}{2}} \cdot \cancel{2}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{1}{\sqrt{2x+1}} \cdot \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{1}{\sqrt{2(4)+1}} \cdot 3$$

$$= \frac{1}{\sqrt{9}} \cdot 3 = \frac{1}{3} \cdot 3 = \boxed{1}$$

Jan 21-10:49 AM

Method II

$$y = \sqrt{2x+1}$$

$$y^2 = 2x+1$$

$$\overset{\checkmark}{2}y\frac{dy}{dt} = \overset{\checkmark}{2}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{1}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{1}{\sqrt{2x+1}}\frac{dx}{dt}$$

Jan 21-10:54 AM

A particle is moving along $xy=8$

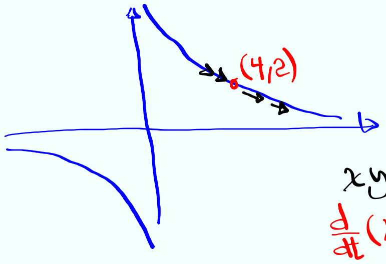
at (4,2) y is decreasing at 3 cm/s.

How fast x is changing?

$\frac{dy}{dt}=-3$

x bigger

$\frac{dx}{dt}>0$



$$xy=8$$

$$\frac{d}{dt}(xy)=\frac{d}{dt}(8)$$

$$\frac{dx}{dt}y+x\frac{dy}{dt}=0$$

$$\frac{dx}{dt}\cdot 2+4\cdot(-3)=0$$

$$\frac{dx}{dt}=\frac{12}{2}=6\text{ cm/s.}$$

Jan 21-10:57 AM

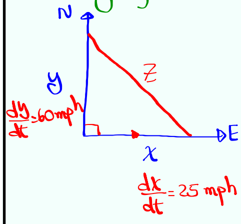
Two cars start moving at the same time.

one going north at 60 mph,

other one going east at 25 mph.

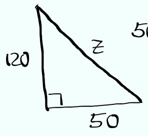
How fast is the distance between them

changing after 2 hrs?



$$x^2+y^2=z^2$$

in 2 hrs



$50^2+120^2=z^2$

$16900=z^2$

$z=130$

$$x^2+y^2=z^2$$

$$\overset{\checkmark}{2x}\frac{dx}{dt}+\overset{\checkmark}{2y}\frac{dy}{dt}=\overset{\checkmark}{2z}\frac{dz}{dt}$$

$$50\cdot 25+120\cdot 60=130\frac{dz}{dt}$$

$$130\frac{dz}{dt}=8450$$

$$\frac{dz}{dt}=\boxed{65\text{ mph}}$$

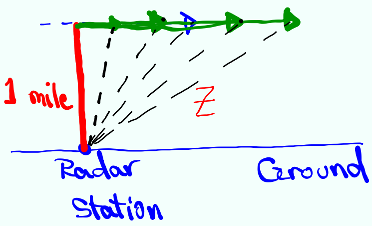
Jan 21-11:02 AM

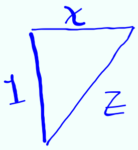
A plane is flying horizontally @ 500 mph.

It passes a radar station at vertical distance of 1 mile.

Distance between plane and radar station is increasing

$$\frac{dz}{dt} > 0$$





$$\frac{dx}{dt} = 500\text{mph}$$

$$x^2 + y^2 = 2^2$$

$$x^2 = 3 \qquad x = \sqrt{3}$$

Find $\frac{dz}{dt}$ when $Z = 2$ miles

$$x^2 + y^2 = z^2$$

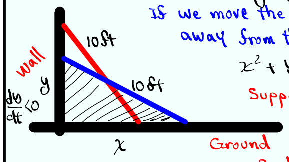
$$\overset{\checkmark}{2x}\frac{dx}{dt} + \overset{\checkmark}{0} = \overset{\checkmark}{2z}\frac{dz}{dt}$$

$$\sqrt{3} \cdot 500 = 2 \cdot \frac{dz}{dt}$$

$$\frac{dz}{dt} = 250\sqrt{3} \text{ mph}$$

Jan 21-11:13 AM

A 10-ft Ladder is leaning against a Wall.



If we move the ladder away from the wall,

$$x^2 + y^2 = 10^2$$

Suppose $\frac{dx}{dt} = 1$ ft/s.

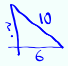
$$\frac{dx}{dt} > 0$$

Find $\frac{dy}{dt}$ when the bottom of ladder is 6ft from the wall.

$$x^2 + y^2 = 10^2$$

$$\overset{\checkmark}{2x}\frac{dx}{dt} + \overset{\checkmark}{2y}\frac{dy}{dt} = 0$$

$$6 \cdot 1 + 8\frac{dy}{dt} = 0$$



$$y^2 + 6^2 = 10^2$$

$$\boxed{y = 8}$$

$$\frac{dy}{dt} = \frac{-6}{8} = \boxed{-.75 \text{ ft/s.}}$$

How fast the angle between ladder and ground changing at that moment?

$$\tan\alpha = \frac{y}{x} \qquad \sin\alpha = \frac{y}{10}$$

$$\sin\alpha = \frac{1}{10}y$$

$$\cos\alpha \cdot \frac{d\alpha}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$$

$$\frac{6}{10} \cdot \frac{d\alpha}{dt} = \frac{1}{10} \cdot \frac{-3}{4}$$

Multiply by 40

$$6 \cdot 4 \frac{d\alpha}{dt} = -3$$

$$\frac{d\alpha}{dt} = \frac{-3}{24} = \frac{-1}{8} \boxed{\text{Rad./s}}$$

Jan 21-11:22 AM

Two Sides of a triangle are 12m & 15m.

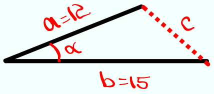
The angle between them is increasing

at the rate of 2°/min.

How fast is the third side changing

when $\boxed{\text{the angle is } 60^\circ}$

$$\frac{d\alpha}{dt} = \overset{\pi/90}{\boxed{2^\circ/\text{min}}}$$



$$c^2 = a^2 + b^2 - 2ab\cos\alpha$$

$$c^2 = 12^2 + 15^2 - 2\cdot12\cdot15\cdot\cos\alpha$$

when $\alpha = 60^\circ$

$$c^2 = 369 - 360\cos 60^\circ$$
$$= 369 - 360\cdot\frac{1}{2}$$
$$= 369 - 180$$
$$= 189$$
$$c = \sqrt{189}$$

$$c^2 = 369 - 360\cos\alpha$$

$$\overset{\checkmark\checkmark}{2c}\,\frac{dc}{dt} = \overset{\checkmark}{-}\overset{\checkmark\checkmark}{360}\cdot\overset{\checkmark}{-}\sin\alpha\cdot\frac{d\alpha}{dt}$$

$$c\,\frac{dc}{dt} = 180\sin\alpha\,\frac{d\alpha}{dt}$$

$$\sqrt{189}\,\frac{dc}{dt} = \overset{2}{\cancel{180}}\cdot\overset{\frac{\sqrt{3}}{\cancel{2}}}{\cancel{\sin}}60^\circ\cdot\frac{\pi}{\cancel{90}}$$

$$\frac{dc}{dt} = \frac{\sqrt{3}\,\pi}{\sqrt{189}} = \pi\sqrt{\frac{3}{189}}$$

$$= \frac{\pi}{\sqrt{63}} \approx \frac{\pi}{8}\ \text{m/min.}$$

Jan 21-11:40 AM

Open Notes

Class QZ II

Use linear approximation to estimate $\sqrt{5}$.

Round to 3-dec. Places.

$f(x) = \sqrt{x}$

$a = 4$

$f(4) = 2$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(4) = \frac{1}{4}$$

$$f(x) \approx f(a) + f'(a)(x-a)$$

$$\sqrt{x} \approx 2 + \frac{1}{4}(x-4)$$

$$\sqrt{5} \approx 2 + \frac{1}{4}(5-4)$$

$$\approx \boxed{2.25}$$

Jan 21-11:53 AM